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Low Energy Dynamics for 1/4 BPS Dyons

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Classical 1/4 BPS configurations consist of 1/2 BPS dyons which are positioned by competing static forces from electromagnetic and Higgs sectors. These forces do not follow the simple inverse square law, but can be encoded in some low energy effective potential between fundamental monopoles of distinct types. In this paper, we find this potential, by comparing the exact 1/4 BPS bound from a Yang-Mills field theory with its counterpart derived from low energy effective dynamics of monopoles. Our method is generalized to arbitrary gauge groups and to arbitrary BPS monopole/dyon configurations. The resulting effective action for 1/4 BPS states is written explicitly, and shown to be determined entirely by the geometry of multi-monopole moduli spaces. We also explore its natural supersymmetric extension.

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1 Introduction

Recently there have been much activity in trying to understand the nature 1/4 BPS dyonic configurations in N=4 supersymmetric Yang-Mills theories [1, 2, 3]. The N=4 supersymmetric theories arise as an effective low energy theory of parallel D3 branes in the type IIB string theory [4]. The expectation values of the six Higgs fields are the coordinates of these D3 branes in the transverse six space. When D3 branes lie on a line, or all Higgs expectation values are aligned in Lie algebra, there can be only 1/2 BPS configurations. Most general 1/2 BPS states are collections of 1/2 BPS dyons whose electric charges are all proportional to the magnetic charge, individually as well as collectively. When the expectation values of the Higgs fields are not aligned, on the other hand, there can be 1/4 BPS configurations, which have a nice string interpretation as multi-pronged strings [5]. As a field theoretic solution, a 1/4 BPS configuration can be thought of as more than one 1/2 BPS dyons at rest with respect each other; the positions of the component dyons are determined by a delicate balance of the electromagnetic Coulomb force and Higgs forces [1]. Because of this, the relative electric charges of distinct type dyons are functions of their relative positions.

The low energy dynamics of 1/2 BPS monopoles has been explored before in many direction, but only in the context of aligned vacua when no static forces among monopoles are possible. The 1/2 BPS configurations are characterized by their moduli parameters, and Manton proposed that the low energy motion of 1/2 BPS monopoles be treated as the geodesic motion on the moduli space[6]. There are several explicitly known moduli space metrics. For a pair of identical monopoles in SU(2) gauge theory, the moduli space is the Atiyah-Hitchin manifold[7]. For a pair of distinct monopoles in SU(3) theory, it is the Taub-NUT manifold [8], and so on.

On the other hand, the low energy dynamics of BPS solitons in misaligned vacua has been more problematic. In the simplest example of SU(3) gauge theory, a 1/4 BPS configuration is known to consist of two 1/2 BPS dyons that are separated by a fixed distance, at which the Coulomb repulsion due to the relative electric charge is exactly balanced against a static attraction induced by Higgs interaction. The explicit form of the potential for the Higgs force, however, has not been well understood.

Recall that the low energy dynamics of BPS solitons explores physics that deviates a little bit from BPS bound. Thus, in certain limit where 1/4 BPS states are almost 1/2 BPS, it should be possible to rediscover physics of 1/4 BPS configurations from the dynamics of 1/2 BPS states. Since we have static forces between 1/2 BPS solitons in misaligned vacua, the simplest possibility is to add a potential term to the moduli space dynamics. In this paper, we find such a low energy effective action that describes both the 1/2 BPS and the 1/4 BPS configurations in misaligned vacua.

The BPS equations for the 1/4 BPS configurations can be grouped into two sets of equations [1]; the first is the old 1/2 BPS equations that produce purely magnetic monopoles, and the second solves for the unbroken global gauge modes in this magnetic background. The solution of the second BPS equation is guaranteed and determines electric charges carried by monopoles of the first BPS equation. Thus, the 1/4 BPS dyons are constructed by dressing 1/2 BPS monopoles electrically, where the amount of the relative electric charge depends on the monopole moduli parameters. A crucial consequence is that the moduli space of monopoles also parameterizes the classical 1/4 BPS dyons but with a twist that some of the parameters characterizes electric charges.

These observations tell us that there are two different ways of constructing 1/4 BPS configurations. The first is to obtain an exact field theoretic classical solution. The energy of such a configuration would saturate the classical BPS bound exactly. On the other hand, since 1/4 BPS dyons are all dressed versions of purely magnetic monopoles, one also should be able to find them as excited charged configurations on the moduli space dynamics. They should also saturate a BPS bound of the low energy effective dynamics. In the limit where the moduli space approximation is good, one then identifies these two BPS bounds, which should constrain the unknown potential term. As a matter of fact, this procedure turns out to be enough to fix the potential completely.

We will first consider the interaction of two distinct dyons in the SU(3) case. We find the exact form of the potential, using the idea outlined above. Furthermore, the resulting Hamiltonian of the low energy dynamics is shown to have a BPS bound. As a consistency check, we show that the configurations that saturate this low energy BPS bound describe the identical physics as the field theoretic 1/4 BPS configurations. We also show that the $1/r$ piece of the potential at large separation is consistent with the results from the study of the interaction of two point-like dyons in large separation.

We then generalize this discussion to any combination of magnetic monopoles in arbitrary gauge group. The form of the potential will turn out to be half the norm of of certain triholomorphic Killing vector field on the moduli space. Here a recent work by D. Tong [9] plays a crucial role. This way, the effective Lagrangian is again determined by the geometry of the moduli space alone. This particular form of potential is known to admit supersymmetric extension, which we also explore.

The plan of the paper is as follows. In Sec.2, we review briefly the 1/4 BPS configuration of two distinct dyons in SU(3) gauge theory and the moduli space metric of these dyons in the

1/2 BPS limit. In Sec.3, we obtain the exact potential for the simplest case of $SU(3)$ when the deviation from the 1/2 BPS case is small, that is, when the D3 branes are almost on a straight line. We show that the BPS configurations of the low energy dynamics are identical to the BPS configuration of the field theoretic ones. In Sec.4, we generalize this result to arbitrary monopole configurations in arbitrary gauge group with more emphasis on the geometrical character of the low energy effective action. The underlying supersymmetric Lagrangian and supercharge, as well as the BPS conditions, are found in Sec. 5. In Sec. 6 we conclude with some remarks.

2 Two Distinct Monopoles in the SU(3) Gauge Theory

The SU(3) gauge group appears in the low energy dynamics of three parallel D3 branes. (See Rf. [1] for detail.) The positions of D3 branes on a plane of the six dimensional transverse direction are dictated by the expectation value of two Higgs fields:

$$\hat{b} \cdot \phi(\infty) = \mathrm{diag}(h_1, h_2, h_3), \tag{1}$$

$$\hat{a} \cdot \phi(\infty) = \xi\ \mathrm{diag}(h_1, h_2, h_3) + \eta\ \mathrm{diag}(\mu_2, -\mu_1 - \mu_2, \mu_1), \tag{2}$$

where $h_1 < h_2 < h_3$, $h_1 + h_2 + h_3 = 0$ and $\mu_1 = h_2 - h_1, \mu_2 = h_3 - h_2$. (Here the string tension is multiplied to the positions of D3 branes so that they acquire the mass dimension.) Two Higgs field expectation values are the coordinates of D3 branes along two orthogonal directions $\hat{b} = (1, 0)$ and $\hat{a} = (0, 1)$. The relative position vector of the second D3 brane with respect to the first D3 brane on the plane is

$$\vec{R}_1 = (\mu_1, \xi\mu_1 - \eta(\mu_1 + 2\mu_2)), \tag{3}$$

while the relative position of the third D3 brane with respect to the second D3 brane is

$$\vec{R}_2 = (\mu_2, \xi\mu_2 + \eta(2\mu_1 + \mu_2)). \tag{4}$$

Two simple roots $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ of the $SU(3)$ groups are chosen in accordance with the convention $\boldsymbol{\alpha}^2 = \boldsymbol{\beta}^2 = 1$, $\boldsymbol{\alpha} \cdot \boldsymbol{\beta} = -1/2$. With $\boldsymbol{\alpha} \cdot \mathbf{H} = \frac{1}{2}\mathrm{diag}(-1, 1, 0)$ and $\boldsymbol{\beta} \cdot \mathbf{H} = \frac{1}{2}\mathrm{diag}(0, -1, 1)$, the masses of isolated $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ monopoles are

$$m_1 = g|\vec{R}_1|, \tag{5}$$

$$m_2 = g|\vec{R}_2|, \tag{6}$$

respectively, where $g = 4\pi$ is the charge of magnetic monopole with $e = 1$ assumed for convenience. Notice that when ξ and η is very small,

$$m_1 = g\mu_1 + \frac{g}{2\mu_1}(\xi\mu_1 - \eta(\mu_1 + 2\mu_2))^2, \tag{7}$$

$$m_2 = g\mu_2 + \frac{g}{2\mu_2}(\xi\mu_2 + \eta(2\mu_1 + \mu_2))^2. \tag{8}$$

There is a third monopole corresponding to the third positive root $\boldsymbol{\alpha} + \boldsymbol{\beta}$ and its mass is $m_3 = g|\vec{R}_1 + \vec{R}_2|$. Contrast to the case when the Higgs vacuum values are aligned, generically there is no distinction between fundamental or composite monopoles. (For example, consider the case where three D3 branes lie on the corners of an equitriangle.) However, when the Higgs vacuum values are almost aligned as in the case we study, we can still distinguish between fundamental and composite monopoles. Thus, $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ monopoles are fundamental and $\boldsymbol{\alpha} + \boldsymbol{\gamma}$ monopoles are composite.

The dyonic configuration we consider is made of one $\boldsymbol{\alpha}$ and one $\boldsymbol{\beta}$ monopoles, with electric charge q_1 and q_2, respectively. Thus the asymptotic forms of the Higgs fields are

$$\hat{b} \cdot \phi = \hat{b} \cdot \phi(\infty) - \frac{1}{4\pi r} g(\boldsymbol{\alpha} + \boldsymbol{\beta}) \cdot \mathbf{H}, \tag{9}$$

$$\hat{a} \cdot \phi = \hat{a} \cdot \phi(\infty) - \frac{1}{4\pi r} (q_1\boldsymbol{\alpha} + q_2\boldsymbol{\beta}) \cdot \mathbf{H}. \tag{10}$$

For the given asymptotic (9), the solution of the first BPS equation

$$B_i = D_i \hat{b} \cdot \phi \tag{11}$$

is uniquely characterized by the relative distance, L, between two monopoles. Once the first BPS solution is found, the solution of the second BPS equation

$$D_i^2 \hat{a} \cdot \phi - [\hat{b} \cdot \phi, [\hat{b} \cdot \phi, \hat{a} \cdot \phi]] = 0 \tag{12}$$

is found to be unique for a given asymptotic (10). From this solution, we can read electric charges

$$q_1 = g(\xi + \eta p_1), \tag{13}$$

$$q_2 = g(\xi + \eta p_2), \tag{14}$$

where

$$p_1 = \frac{\mu_1 - \mu_2 - 2(\mu_1 + 2\mu_2)\mu_2 L}{\mu_1 + \mu_2 + 2\mu_1\mu_2 L}, \tag{15}$$

$$p_2 = \frac{\mu_1 - \mu_2 + 2(2\mu_1 + \mu_2)\mu_1 L}{\mu_1 + \mu_2 + 2\mu_1\mu_2 L}. \tag{16}$$

As discussed in the introduction, one of the interesting things about these BPS solutions is that we may treat the 1/4 BPS configurations as if they are made of 1/2 BPS monopoles with some electric dress. The first BPS equation can be regarded as the 1/2 BPS equation. Its solution describes a collection of 1/2 BPS monopoles and is characterized uniquely by the moduli space coordinates. Here the constituent monopoles carry mass characterized by the Higgs asymptotic value $\hat{b} \cdot \phi(\infty)$. Then we solve the second BPS equation, which is identical to the gauge zero mode equation of the first BPS equation. The solutions of the second BPS equation is determined uniquely by the moduli parameters of the solution of the first BPS equation and by the value $\hat{a} \cdot \phi(\infty)$. One of the key point of this paper is to take this view further and to regard the low energy dynamics of 1/4 BPS dyons as that of 1/2 BPS monopoles described by the first BPS equation.

Thus, instead of m_1 and m_2, we regard $g\mu_1$ and $g\mu_2$ as the real mass of monopoles. We can define the total and relative charges with respect to the masses $g\mu_1$ and $g\mu_2$, that is,

$$q_{\rm tot} = \frac{\mu_1 q_1 + \mu_2 q_2}{\mu_1 + \mu_2} = g\left(\xi + \eta\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right), \tag{17}$$

$$\Delta q(L) = \frac{q_2 - q_1}{2} = \frac{\Delta q_c}{1 + \frac{1}{2\mu L}}, \tag{18}$$

where the critical relative charge is defined as

$$\Delta q_c \equiv g\eta\frac{\mu_1^2 + \mu_1\mu_2 + \mu_2^2}{\mu_1\mu_2}, \tag{19}$$

and the pseudo relative mass is

$$\mu = \frac{\mu_1\mu_2}{\mu_1 + \mu_2}. \tag{20}$$

Notice that the relative charge vanishes when $L = 0$ and tends to Δq_c at $L = \infty$. (When three D3 branes lie on a line and so $\eta = 0$, the relative charge vanishes.) We further note that the mass difference, $\sum_i (m_i - g\mu_i)$, may be written, in terms of charges, as

$$m_1 + m_2 - g(\mu_1 + \mu_2) = \frac{\mu_1 + \mu_2}{2g}(q_{\rm tot})^2 + \frac{2\mu}{g}(\Delta q_c)^2. \tag{21}$$

The difference between m_i and $g\mu_i$ can be considered as a result of the interaction between monopoles, which even includes a constant potential.

2.1 BPS Energy

In the BPS energy bound, it is natural to introduce the two-dimensional magnetic and electric charge vectors,

$$\vec{Q}^M = g(\vec{R}_1 + \vec{R}_2), \tag{22}$$

$$\vec{Q}^E = q_1\vec{R}_1 + q_2\vec{R}_2. \tag{23}$$

The central charge of the N=4 supersymmetric algebra, which gives the BPS energy bound, is

$$Z_{\pm}^2 = \max\left(\vec{Q}_M^2 + \vec{Q}_E^2 \pm 2\vec{Q}_E \times \vec{Q}_M\right). \tag{24}$$

When the BPS equations are satisfied so that the BPS energy bound is saturated, there is another expression for the central term Z. Since $\hat{a}\cdot\vec{Q}^M = \hat{b}\cdot\vec{Q}^E$, the energy is

$$\begin{aligned} Z_+ &= \hat{b}\cdot\vec{Q}^M + \hat{a}\cdot\vec{Q}^E \\ &= g(\mu_1+\mu_2) + \frac{\mu_1+\mu_2}{g}(q_{\rm tot})^2 + \frac{4\mu}{g}\Delta q(L)\Delta q_c, \end{aligned} \tag{25}$$

which is exact. As $\hat{b}\cdot\vec{Q}^M = g(\mu_1+\mu_2)$ is just the sum of constituent monopole masses, we regard the rest of the contribution, $\hat{a}\cdot\vec{Q}^E$ to arise from the dynamics of monopoles.

The low energy dynamics means that the dynamical energy contribution to the rest mass is small. From Eqs. (17) and (18), we see that the low energy approximation may holds if

$$|\xi|, \;\; |\eta| \;<< 1. \tag{26}$$

From Eqs. (1) and (2), we see that three D3 branes are almost collinear, for the low energy approximation to hold. The above condition also implies that the magnitude of $q_{\rm tot}, \Delta q$ of 1/4 BPS configurations should be very small compare with the magnetic charge g. From our point of view, we regard $\hat{a}\cdot\vec{Q}^E$ in the central charge arises from the low energy dynamics of monopoles. As we will see, it has the contributions from both kinetic and potential energies.

2.2 Moduli Space Metric of 1/2 BPS Monopoles

When three D3 branes are collinear, the low energy dynamics of two monopoles can be described by the moduli space or collective coordinate dynamics. There are four zero modes for each monopole, three for its position and one for the U(1) phase. We call their positions and phases to be $\mathbf{x}_i, \psi_i, i = 1,2$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ monopoles, respectively. The exact nonrelativistic effective Lagrangian has been found to be a sum of the Lagrangians for the center of mass and the relative motion [8]. As there is no external force, the center of mass Lagrangian is free one,

$$\mathcal{L}_{\rm cm} = \frac{g(\mu_1+\mu_2)}{2}\dot{\mathbf{X}}^2 + \frac{g}{2(\mu_1+\mu_2)}\dot{\chi}^2, \tag{27}$$

where the center of mass position is $\mathbf{X} = (\mu_1\mathbf{x}_1 + \mu_2\mathbf{x}_2)/(\mu_1+\mu_2)$ and the center of mass phase is $\chi = \psi_1 + \psi_2$. The relative motion between them is more complicated and described by the Taub-NUT metric,

$$\mathcal{L}_{\rm rel} = \frac{g\mu}{2}\left((1+\frac{1}{2\mu r})\dot{\mathbf{r}}^2 + \frac{1}{4\mu^2(1+\frac{1}{2\mu r})}(\dot{\psi} + \mathbf{w}(\mathbf{r})\cdot\dot{\mathbf{r}})^2\right), \tag{28}$$

where the relative position $\mathbf{r} = \mathbf{x}_2 - \mathbf{x}_1$, the relative phase is $\psi = 2(\mu_1\psi_2 - \mu_2\psi_1)/(\mu_1+\mu_2)$, and $\mathbf{w}(\mathbf{r})$ is the Dirac potential such that $\nabla \times \mathbf{w}(\mathbf{r}) = -\mathbf{r}/r^3$. The range of ψ is $[0, 4\pi]$. The relative moduli space $\mathcal{M}_0$ is the Taub-NUT manifold with the metric given from the above Lagrangian. The eight dimensional moduli space is then given by

$$\mathcal{M} = R^3 \times \frac{S^1 \times \mathcal{M}_0}{Z}, \tag{29}$$

where Z is the identification map

$$(\chi, \psi) = (\chi + 2\pi, \psi + \frac{4\pi\mu_2}{\mu_1+\mu_2}), \tag{30}$$

One way of obtaining this moduli space metric is by exploring the long range interaction between two dyons[10]. As for two dyons of charge $(g, \pi_1)\boldsymbol{\alpha}$ and $(g, \pi_2)\boldsymbol{\beta}$, there are several interactions between them. Obvious ones are electric and magnetic Coulomb potentials. Besides, there exists a potential due to Higgs force between them.. In addition when they move, there is a time delay effect which appears in the form of Lienard-Wiechert potential. When they are put together, we get the Routhian obtained from the Legendre transformation of phase variables to charges,

$$\mathcal{R} = \frac{g(\mu_1+\mu_2)}{2}\left(\dot{\mathbf{X}}^2 - \frac{(\pi_{\rm t})^2}{g^2}\right) + \frac{g\mu}{2}(1+\frac{1}{2\mu r})\left(\dot{\mathbf{r}}^2 - \frac{4(\pi_{\rm r})^2}{g^2}\right) + \pi_{\rm r}\mathbf{w}(\mathbf{r})\cdot\dot{\mathbf{r}}, \tag{31}$$

where the relation between $\pi_{\rm t}, \pi_{\rm r}$ and π_1 and π_2 are given by similar equations as in Eqs. (17) and (18). We emphasize here that total charge $\pi_{\rm t}$ and relative charge $\pi_{\rm r}$ are conjugate momenta without any fixed value.

3 Case of $SU(3)$

To find the potentials between two dyons in every relative separation, we explore first the case $q_{tot} = \pi_{\rm t} = 0$. Suppose that the exact potential for the relative motion is $\mathcal{U}_{\rm rel}(r)$, and so the effective potential for two dyons is

$$U_{\rm eff}(r) = \frac{2\mu}{g}\left(1+\frac{1}{2\mu r}\right)(\pi_{\rm r})^2 + \mathcal{U}_{\rm rel}(r) \tag{32}$$

The first part of the effective potential comes from the charge kinetic energy. This effective potential should have a minimum at r if $\pi_{\rm r} = \Delta q(r)$ of Eq. (18) and the energy at the minimum should be identical to the BPS energy (25) once we add the mass of monopoles. Then we realize that the potential energy is exactly half of the excessive BPS energy associated with the relative charge. Using this, we find the piece of the potential,

$$\mathcal{U}_{\rm rel}(r) \quad = \quad \frac{2\mu}{g}\frac{(\Delta q_c)^2}{1+\frac{1}{2\mu r}}$$

$$= \frac{2\mu}{g}(1+\frac{1}{2\mu r})(\Delta q(r))^2 \tag{33}$$

where $\Delta q(r)$ is understood as functions of r given in (18). However, we have not quite found the potential $\mathcal{U}$. We actually found only the piece that depends on the distance r. By including the center of mass motion, it is not difficult to guess that the actual potential must be of the form,

$$\mathcal{U}(r) = \frac{\mu_1+\mu_2}{2g}q_{\rm tot}^2 + \frac{2\mu}{g}\frac{(\Delta q_c)^2}{1+\frac{1}{2\mu r}} \tag{34}$$

This guess will be justified below momentarily.

As we have seen, this identification can be made for all possible classical dyons, which translates to all possible values of r. In effect, we have found the potential $\mathcal{U}$ throughout the moduli space. In the next section, we will discover that this method is trivially generalized to arbitrary configurations of monopoles and dyons in arbitrary gauge group, and that it produces a rather special kind of potential, which, as one may expect, admits supersymmetric extension.

Let us go back to the matter of total charge and its associated constant potential energy below. If we minimize the effective potential (32), we get π_r to be $\Delta q(r) = \Delta q_c/(1+\frac{1}{2\mu r})$. However, we could not fix π_t to be q_{tot} in this way. We get that $\pi_{\rm t}$ to be constant in time. Thus, the naive equilibrium condition does not fix the total electric charge, even though it fixes the relative charge. To understand this, let us now collect the full low energy Lagrangian for 1/4 BPS dyons, which is the sum of the kinetic energies (27), (28) on the moduli space and the potential energy (34)

$$\mathcal{L}_{\rm low} = \mathcal{L}_{\rm cm} + \mathcal{L}_{\rm rel} - \mathcal{U}(r) \tag{35}$$

In terms of the conjugate momenta, $\mathbf{P}_{\rm t} = g(\mu_1+\mu_2)\dot{\mathbf{X}}$, $\mathbf{p} = g\mu(1+\frac{1}{2\mu r})\dot{\mathbf{r}} + \pi_{\rm r}\mathbf{w}(\mathbf{r})$, $\pi_{\rm t}$, and $\pi_{\rm r}$, the Hamiltonian is

$$\begin{aligned}\mathcal{H} &= \frac{1}{2g(\mu_1+\mu_2)}\mathbf{P}^2 + \frac{1}{2g\mu}\frac{1}{1+\frac{1}{2\mu r}}(\mathbf{p}-\pi_{\rm r}\mathbf{w}(\mathbf{r}))^2 \\ &\quad + \frac{\mu_1+\mu_2}{2g}(\pi_t)^2 + \frac{2\mu}{g}(1+\frac{1}{2\mu r})\pi_{\rm r}^2 + \mathcal{U}(r).\end{aligned} \tag{36}$$

The Hamiltonian can be expressed as follows:

$$\begin{aligned}\mathcal{H} &= \frac{1}{2g(\mu_1+\mu_2)}\mathbf{P}^2 + \frac{1}{2g\mu}\frac{1}{1+\frac{1}{2\mu r}}(\mathbf{p}-\pi_{\rm r}\mathbf{w}(\mathbf{r}))^2 \\ &\quad + \frac{\mu_1+\mu_2}{2g}(\pi_t \mp q_{tot})^2 + \frac{2\mu}{g}(1+\frac{1}{2\mu r})(\pi_{\rm r} \mp \Delta q(r))^2 \pm \mathcal{Z},\end{aligned} \tag{37}$$

where the new central term is

$$\mathcal{Z} = \frac{\mu_1+\mu_2}{g}q_{\rm tot}\pi_{\rm t} + \frac{4\mu}{g}\Delta q_c \pi_{\rm r}. \tag{38}$$

The central term is linear in the conjugate momenta $\pi_{\rm t}$ and $\pi_{\rm r}$. Clearly there is a classical BPS bound on the mechanical energy

$$\mathcal{H} \geq |\mathcal{Z}|, \tag{39}$$

which is saturated for any $\mathbf{X}$ and $\mathbf{r}$ when $\pi_{\rm t} = q_{\rm tot}$ and $\pi_{\rm r} = \Delta q(r)$ and $\dot{\mathbf{X}} = \dot{\mathbf{r}} = 0$. Thus the nonrelativistic BPS configuration matches exactly the field theoretic BPS configuration. In this case, the sum of the rest mass plus this BPS energy is exactly the 1/4 BPS energy bound (25). Thus our 1/4 BPS field configuration corresponds to not the lowest energy configuration, but a BPS saturated configuration of the nonrelativistic Hamiltonian.

As an independent check of the above potential, let us consider the leading term in the large distance. When three D3 branes are not collinear, two Higgs fields are involved nontrivially and so we expect an additional Higgs interaction [11]. To see this, let us go back to the old derivation of the Higgs interaction. We put the resting $\boldsymbol{\alpha}$ dyon at $\mathbf{x}_1$. The Higgs field far from this dyon is

$$\phi_I(\mathbf{x}) = \phi_I(\infty) - \frac{\hat{R}_{1I}}{4\pi|\mathbf{x}-\mathbf{x}_1|}\boldsymbol{\alpha}\cdot\mathbf{H}\sqrt{g^2+\pi_1^2}, \tag{40}$$

in the unitary gauge. Note that $\boldsymbol{\alpha}\cdot\phi_I(\infty) = \vec{R}_{1I}$. The $\boldsymbol{\beta}$ monopole at $\mathbf{x} = \mathbf{x}_2$ would feel this Higgs field as an effective mass

$$-m_{\rm eff} = -\sqrt{g^2+\pi_2^2}\left|\vec{R}_2 - \hat{R}_1\sqrt{g^2+\pi_1^2}\,\frac{\boldsymbol{\alpha}\cdot\boldsymbol{\beta}}{4\pi r}\right|, \tag{41}$$

where $r = |\mathbf{x}_2 - \mathbf{x}_1|$.

We expand the effective mass to order $1/r$ and quadratic in π_1 and π_2 to get

$$-m_{\rm eff} = -\left(1+\frac{\pi_2^2}{2g^2}\right)m_2 - \frac{g^2}{8\pi r}\hat{R}_1\cdot\hat{R}_2\left(1+\frac{\pi_1^2}{2g^2}\right)\left(1+\frac{\pi_2^2}{2g^2}\right). \tag{42}$$

If three D3 branes are collinear, $\hat{R}_1 = \hat{R}_2$ and we get the old result. When we include the velocity dependent terms and electromagnetic forces and keep everything in quadratic order, we get the previous Routhian (31).

However, when they are not collinear, even though very close, there is an additional $1/r$ correction to the old result. As we assume that the deviation from the straight line is very small, or $\eta << 1$,

$$\hat{R}_1\cdot\hat{R}_2 = \cos\theta \approx 1 - \frac{\theta^2}{2}. \tag{43}$$

From Eqs. (3) and (4), one can show easily that

$$\begin{aligned} \theta &= 2\eta\frac{\mu_1^2+\mu_1\mu_2+\mu_2^2}{\mu_1\mu_2} + \mathcal{O}(\eta^2) \\ &= \frac{2\Delta q_c}{g} \end{aligned} \tag{44}$$

with Δq_c in Eq. (19). The small deviation obtained from Eq. (42) is then

$$\begin{aligned}\Delta L &= -m_1 - m_2 + g(\mu_1+\mu_2) + \frac{g^2}{8\pi r}(1-\hat{R}_1\cdot\hat{R}_2) \\ &= -\frac{\mu_1+\mu_2}{2g}(q_{\rm tot})^2 - \frac{2\mu}{g}(\Delta q_c)^2 + \frac{(\Delta q_c)^2}{4\pi r},\end{aligned} \tag{45}$$

where we have included the additional constant terms from masses that are given explicitly in (21). Here we have dropped terms of order $(\Delta q_c)^2(\pi_i)^2$, which are much smaller. This is the additional attractive potential due to the fact that three D3 branes are not collinear. It is a matter of simple algebra to check that the exact potential (34) we found above, do contain this term to the leading r-dependent piece at large r. One may wonder about terms of order $1/r^2$. These terms does not comes out to be symmetric under exchange of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ monopoles and depends on the monopole core size. It needs a further investigation, which we do not attempt here.

4 Generalization

The crucial ingredient of the preceding section came from a rather simple observation; we have two different ways of estimating the 1/4 BPS dyon mass. The field theoretic one gives an exact expression, while the other is derived from approximate low energy dynamics on the moduli space of monopoles. This is possible because the moduli space dynamics already incorporate various internal excitations that induce relative electric charges. By comparing the two, one obtains an approximate form of the inter-monopole potential that is arbitrarily accurate as the Higgs expectations become collinear. In this section, we will utilize this simple idea to fix the bosonic part of low energy effective Lagrangian for all 1/2 BPS and 1/4 BPS states in maximally supersymmetric Yang-Mills theory.

First consider a 1/2 BPS multi-monopole configuration, when an arbitrary gauge group G is broken to $U(1)^r$ by a single Higgs field expectation value $\langle \hat{b}\cdot\phi\rangle = \mathbf{h}$ in the root space. There are always a set of simple roots $\boldsymbol{\beta}_A, A=1,...r$ such that $\mathbf{h}\cdot\boldsymbol{\beta}_A > 0$. Any BPS magnetic monopole configuration would have the magnetic charge

$$\mathbf{g} = g(n_1\boldsymbol{\beta}_1 + n_2\boldsymbol{\beta}_2 + ... + n_r\boldsymbol{\beta}_r), \tag{46}$$

with nonnegative $n_A's$. Without loss of generality, we assume that it is irreducible so that all n_A are positive. In this case all monopoles are interacting each other directly or indirectly and are not divided into noninteracting subclusters.

Each monopole of $\boldsymbol{\beta}_A$ type has mass $g\mu_A = g\mathbf{h}\cdot\boldsymbol{\beta}_A$ and four zero modes. The total energy is then $g\sum_A n_A\mu_A = \mathbf{g}\cdot\mathbf{h}$ and the total number of zero mode is $4N = 4\sum_A n_A$ [12]. The low energy dynamics of this configuration can be described by the dynamics of the moduli space of dimension $4N$ with the metric $ds^2 = g_{\mu\nu} dz^\mu dz^\nu$. This moduli space is known to be hyperkähler that are equipped with three covariantly constant complex structures. The low energy Lagrangian is then

$$\mathcal{L}_1 = \frac{1}{2}\, g_{\mu\nu} \dot{z}^\mu \dot{z}^\nu . \tag{47}$$

The unbroken $U(1)^r$ symmetries act on the moduli space as translational isometries, and generate cyclic $U(1)$ coordinate ψ^A's, whose conjugate momenta q_A's are conserved electric charges. We divide the moduli space coordinates z^μ's into r ψ^A's and $(4N-r)$ y^i's. Up to gauge transformations, the solution of the first BPS equation (11) are uniquely characterized by the values of the coordinates y^i. The Lagrangian (47) can be rewritten as

$$\mathcal{L}_1 = \frac{1}{2}\, h_{ij} \dot{y}^i \dot{y}^j + \frac{1}{2} L_{AB} (\dot{\psi}^A + w^A_i \dot{y}^i)(\dot{\psi}^B + w^B_j \dot{y}^j). \tag{48}$$

Due to the cyclic properties of ψ^A's, the metric components h_{ij}, L_{AB} and w^A_i are functions of y^i only. Since the kinetic energy should be positive, the metric h_{ij} and L_{AB} are positive definite. Each $U(1)$ generators are associated with the vector field $K_A = \partial/\partial\psi^A = \delta^\mu_A \partial/\partial z^\mu$, which is a (triholomorphic) Killing vector field. Finally, note that $L_{AB}(y) = g_{\mu\nu} K^\mu_A K^\nu_B$.

Denoting the conjugate momenta as

$$q_A = L_{AB}(\dot{\psi}^B + w^B_j \dot{y}^j), \tag{49}$$

$$p_i = h_{ij} \dot{y}^j + q_A w^A_i, \tag{50}$$

the electric charge of the whole configuration is expressed in terms of the q_A's as

$$\mathbf{q} = q_1 \boldsymbol{\beta}_1 + q_2 \boldsymbol{\beta}_2 + ... + q_r \boldsymbol{\beta}_r . \tag{51}$$

The Hamiltonian $\mathcal{H}_1 = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$ is

$$\mathcal{H}_1 = \frac{1}{2} h^{ij} (p_i - q_A w^A_i)(p_j - q_B w^B_j) + \frac{1}{2} (L^{-1})^{AB} q_A q_B . \tag{52}$$

Since L^{-1} depends on the y^i's, this shows that exciting electric charges will typically induce a long range potential. Because of this, 1/4 BPS configurations of dyons with relative charge cannot be allowed if is no other static forces are present, as would be the case if Higgs expectations were all aligned.

Suppose that the other Higgs field $\hat{a} \cdot \phi$ acquires an expectation value which is misaligned with respect to $\langle \hat{b} \cdot \phi \rangle$. Then there is attractive static force between monopoles. If the misalignment is small enough so that the static attractive potential is much smaller than monopole mass scale, we may incorporate this potential to the above moduli space dynamics. (Part of Higgs expectation $\hat{a}\cdot\phi$ that is proportional to that of $\hat{b}\cdot\phi$ is not associated with any static force. Rather, its effect on the BPS configuration is to fix the total charge to a certain value, and to give an additional contribution to the energy besides the energy carried by the charge. This can be understood as the correction to the bare mass of magnetic monopoles.)

Adding a potential to the Hamiltonian and considering the static configuration with $\dot{y}_i = 0$, we get an effective potential

$$U_{\text{eff}} = \frac{1}{2} q_A (L^{-1}(y))^{AB} q_B + \mathcal{U}(y). \tag{53}$$

For 1/4 BPS configurations, q_A should be determined by the moduli parameters y^i, or reversely, given values of q_A's restrict the equilibrium positions y^i.

On the other hand, an exact expression of the 1/4 BPS configuration is known. The additional energy due to the electric charge is given by a simple expression,

$$E_Q = \hat{a}\cdot\vec{Q}^E = a^A \bar{q}_A(y). \tag{54}$$

The second equality defines the projected Higgs expectation values a^A. The electric charge are fixed by the positions of magnetic monopoles, which we emphasize by introducing new notation $\bar{q}_A(y)$. A recent work by D. Tong [9] brought some light on this quantity. He expressed $\bar{q}_A(y)$ in terms of quantities on moduli space and found,

$$\bar{q}_A(y) \equiv L_{AB}(y)\, a^B. \tag{55}$$

This equation can be viewed in two equivalent ways. One is as a restriction on the possible equilibrium position y^i when the electric charge is given. Or equivalently, as an expression of the electric charges in terms of the equilibrium position y^i. Either way, the excess energy due to the electric charge excitation is

$$a^A \bar{q}_A(y) = a^A L_{AB}(y)\, a^B = \bar{q}_A(y)\, (L^{-1}(y))^{AB} \bar{q}_B(y). \tag{56}$$

Note that this happens to be twice the charge kinetic energy if we put $q_A = \bar{q}_A(y)$. Again we propose the potential $\mathcal{U}(y)$ to be identical to the half of E_Q and to be, when expressed as a function of y^i,

$$\mathcal{U}(y) = \frac{1}{2}\, a^A L_{AB}(y)\, a^B. \tag{57}$$

We need to pause here for a moment, and explain how it was possible that we obtained the potential $\mathcal{U}$ when we only considered the classical minimum energy configurations of U_{eff}. It may seem that we made an extrapolation of some kind. However, this is not the case. We actually have derived the exact potential, as we will see.

Why is it so? From Ref. [1], we know that solutions to the first BPS equation (11) are characterized by moduli parameters y^i's. That is, any given set of y^i, a purely magnetic solution exists. Then one solves the second BPS equation (12), which leads to 1/4 BPS dyons whose configuration satisfies the relation $q_A = \bar{q}_A(y) = L_{AB}(y)a^B$. In other words, there are 1/4 BPS configurations for any generic values of y^i's. Therefore, while we identified the value of potential $\mathcal{U}$ for individual 1/4 BPS states, we actually learn the values $\mathcal{U}$ at all y^i by considering all possible classical 1/4 BPS dyons.

Here, we need to make one final consistency check. Not only the value of the potential $\mathcal{U}(y)$ at the 'minimum' of the effective potential $U_{\rm eff}(y)$ should yield the right value, which led to the above identification, but also it should have the 'minimum' at the right value. In other words, we must recover the central relationship, $q_A = \bar{q}_A(y) = L_{AB}(y)a^B$ of field theory origin, by minimizing the low energy dynamics. The final Hamiltonian with the potential is

$$\mathcal{H} = \frac{1}{2}h_{ij}(y)\dot{y}^i\dot{y}^j + \frac{1}{2}(L^{-1})^{AB}(y)q_A q_B + \frac{1}{2}a^A L_{AB}(y)a^B, \tag{58}$$

where $\dot{y}^i = h^{ij}(p_j - q_A w^A_j)$. As in the previous section, we can re-express this as

$$\mathcal{H} = \frac{1}{2}h_{ij}(y)\dot{y}^i\dot{y}^j + \frac{1}{2}(L^{-1})^{AB}(y)(q_A \mp \bar{q}_A(y))(q_B \mp \bar{q}_B(y)) \pm \mathcal{Z}, \tag{59}$$

where the central charge is

$$\mathcal{Z} = \bar{q}_A(y)\,(L^{-1})^{AB}(y)\,q_B = a^A\,q_A. \tag{60}$$

The BPS bound is saturated when $\dot{y}^i = 0$ and $q_A = \bar{q}_A(y)$, exactly as we have hoped for. This completes the derivation of the potential $\mathcal{U}$.

The effective Lagrangian for the low energy is the sum of the usual kinetic term on the moduli space and the potential $\mathcal{U}$,

$$\mathcal{L} = \frac{1}{2}\,h_{ij}\dot{y}^i\dot{y}^j + \frac{1}{2}L_{AB}(\dot{\psi}^A + w^A_i\dot{y}^i)(\dot{\psi}^B + w^B_j\dot{y}^j) - \frac{1}{2}a^A L_{AB}(y)a^B, \tag{61}$$

or more compactly,

$$\mathcal{L} = \frac{1}{2}\,g_{\mu\nu}\dot{z}^\mu\dot{z}^\nu - \frac{1}{2}\,g_{\mu\nu}(a\cdot K)^\mu(a\cdot K)^\nu. \tag{62}$$

Note that, given the Higgs expectation values, the potential is completely determined in terms of geometry of the monopole moduli space. As showed by Alvarez-Gaume and Freedman [13], and as pointed out by D. Tong [9] very recently, this form of potential is exactly what one needs to extend the dynamics to a supersymmetric one. In particular, the triholomorphicity of $a\cdot K$ that follows from gauge invariance, guarantees that the low energy dynamics will have 8 real supercharges. In this sense the dynamics itself is 1/2 BPS with respect to the Yang-Mills field theory. The quantum counterpart of the classical 1/4 BPS dyons should break additional half of these remaining 8 supercharges, and is realized as finite energy BPS states of this low energy theory itself. In the next section we will explore this supersymmetric dynamics in some detail.

5 Supersymmetry and BPS Bound

We begin with the N=4 supersymmetric quantum extension of the above effective action [13]. Its form is similar to the usual sigma model action but modified by a coupling to the triholomorphic Killing vector $G \equiv a \cdot K$. The supersymmetric Lagrangian written with real fermions is

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\Big(g_{\mu\nu}\dot{z}^\mu \dot{z}^\nu + i g_{\mu\nu}\bar{\psi}^\mu \gamma^0 D_t \psi^\nu + \frac{1}{6} R_{\mu\nu\rho\sigma}\bar{\psi}^\mu\psi^\rho\bar{\psi}^\nu\psi^\sigma \\ & \quad - g^{\mu\nu} G_\mu G_\nu - D_\mu G_\nu \bar{\psi}^\mu \gamma_5 \psi^\nu \Big), \end{aligned} \tag{63}$$

where ψ^μ is a two-component anticommuting Majorana spinor and $\gamma^0 = \sigma_2$, $\gamma_5 = \sigma_3$, and $\bar{\psi} = \psi^T \gamma^0$. The metric here is hyperkähler endowed with three complex structures $f^{(a)\mu}{}_\nu (a = 1, 2, 3)$ that satisfy

$$f^{(a)} f^{(b)} = -\delta^{ab} + \epsilon^{abc} f^{(c)} \tag{64}$$

$$D_\mu f^{(a)\nu}{}_\rho = 0\,, \tag{65}$$

and the Killing vector G^μ is triholomorphic, i.e., its action preserves all three complex structures, $f^{(a)}$. From now on, we will use f to denote any one of the three complex structures, unless noted otherwise.

With vielbein e^A_μ and the spinors $\psi^A = e^A_\mu \psi^\mu$, we define supercovariant momenta by

$$\pi_\mu \equiv p_\mu - \frac{i}{2}\omega_{AB\mu}\bar{\psi}^A\gamma^0\psi^B \tag{66}$$

where the p's are canonical momenta of coordinate z's, and $\omega^A{}_{B,\mu}$ is the spin connection. The canonical commutation relations are $[z^\mu, p_\nu] = i\delta^\mu_\nu$ and $\{\psi^A_\alpha, \psi^B_\beta\} = \delta^{AB}\delta_{\alpha\beta}$. SUSY generators in real form are:

$$Q_\alpha = \psi^\mu_\alpha \pi_\mu + i(\gamma^0\gamma_5\psi^\mu)_\alpha G_\mu, \tag{67}$$

$$Q^f_\alpha = f^\mu{}_\nu \psi^\nu_\alpha \pi_\mu + i(\gamma^0\gamma_5 f^\mu{}_\nu \psi^\nu)_\alpha G_\mu, \tag{68}$$

which satisfy the following SUSY algebra:

$$\{Q_\alpha, Q_\beta\} = \{Q^f_\alpha, Q^f_\beta\} = 2\delta_{\alpha\beta}\,\mathcal{H} + 2i(\gamma^0\gamma_5)_{\alpha\beta}\,\mathcal{Z} \tag{69}$$

$$\{Q_\alpha, Q^f_\beta\} = 0 \tag{70}$$

Similarly, supercharges associated with different complex structures $f^{(a)}$ anticommute. The Hamiltonian, $\mathcal{H}$, and the central charge, $\mathcal{Z}$, read

$$\mathcal{H} = \frac{1}{2}\Big(\frac{1}{\sqrt{g}}\pi_\mu\sqrt{g}g^{\mu\nu}\pi_\nu + G_\mu G^\mu - \frac{1}{4}R_{\mu\nu\rho\sigma}\bar{\psi}^\mu\gamma^0\psi^\nu\bar{\psi}^\rho\gamma^0\psi^\sigma + D_\mu G_\nu\bar{\psi}^\mu\gamma_5\psi^\nu\Big) \tag{71}$$

$$\mathcal{Z} = G^\mu\pi_\mu - \frac{i}{2}(D_\mu G_\nu)\bar{\psi}^\mu\gamma^0\psi^\nu \tag{72}$$

It is checked easily that the central charge $\mathcal{Z}$ indeed commutes with all SUSY generators.

For spectrum analysis, SUSY generators in complex form are more useful. Introducing $\varphi \equiv \frac{1}{\sqrt{2}}(\psi^\mu_1 - i\psi^\mu_2)$, and defining $Q \equiv \frac{1}{\sqrt{2}}(Q_1 - iQ_2)$, one finds

$$Q = \varphi^\mu\pi_\mu + i\varphi^{*\mu}G_\mu, \tag{73}$$

$$Q^\dagger = \varphi^{*\mu}\pi_\mu - i\varphi^\mu G_\mu, \tag{74}$$

which generates the following simple algebra:

$$\{Q, Q^\dagger\} = \{Q^f, Q^{f\dagger}\} = 2\mathcal{H}, \tag{75}$$

$$\{Q, Q\} = \{Q^f, Q^f\} = -\{Q^\dagger, Q^\dagger\} = -\{Q^{f\dagger}, Q^{f\dagger}\} = 2i\mathcal{Z}, \tag{76}$$

$$\{Q, Q^f\} = \{Q^\dagger, Q^f\} = 0. \tag{77}$$

Again, supercharges associated with different complex structures $f^{(a)}$ anticommute.

It is easy to read out the BPS condition for quantum states that preserves half of supersymmetries. Depending on the sign of central charge, we find

$$(Q \mp iQ^\dagger)|\Phi\rangle = 0, \tag{78}$$

that saturates $\mathcal{H} = \pm\mathcal{Z}$. We can express this BPS condition more geometrical fashion by transcribing the wavefunction to differential forms on the moduli space. Note that

$$[i\pi_\mu, \varphi^\nu] = -\Gamma^\nu_{\mu\rho}\varphi^\rho, \tag{79}$$

$$[i\pi_\mu, \varphi^*_\nu] = \Gamma^\rho_{\mu\nu}\varphi^*_\rho, \tag{80}$$

$$\{\varphi^\mu, \varphi^*_\nu\} = \delta^\mu_\nu. \tag{81}$$

Furthermore, the wavefunction has the following general form,

$$|\Phi\rangle = \sum_p \frac{1}{p!}\,\Omega_{\mu_1\cdots\mu_p}(z^\mu)\varphi^{\mu_1}\cdots\varphi^{\mu_p}|0\rangle \tag{82}$$

$$\varphi^{*\mu}|0\rangle = 0. \tag{83}$$

The coefficients $\Omega_{\mu_1\cdots\mu_p}$ are completely antisymmetric, and may be regarded as those of a p-form. In this language, we interpret φ^μ and φ^*_μ as a natural cobasis dz^μ and a natural basis $\frac{\partial}{\partial z^\mu}$, one finds that the following replacement is equivalent:

$$i\varphi^\mu\pi_\mu \to d\,, \quad i\varphi^{*\mu}\pi_\mu \to -\delta, \tag{84}$$

$$\varphi^{*\mu}G_\mu \to i_G\,, \quad i\mathcal{Z} \to \mathcal{L}_G \equiv di_G + i_G d, \tag{85}$$

where i_G denotes the natural contraction of the vector field G with a differential form. The BPS equation now becomes

$$*(id + i_G)\,\Omega = \pm\, i(id + i_G) * \Omega \tag{86}$$

where $*$ is the Hodge dual operator. Solving this first order system, we should recover all 1/2 BPS and 1/4 BPS states of the underlying Yang-Mills field theory. Work is currently in progress to solve this equation in the simplest case of $SU(3)$ [15].

6 Conclusion

We have found the low energy effective Lagrangian of 1/2 BPS monopoles in vacua of misaligned Higgs expectation values. This low energy effective theory produces 1/4 BPS dyons as BPS configurations of the nonrelativitic Hamiltonian. The kinetic term is given by the usual moduli space metric of 1/2 BPS monopoles, while the potential term is also determined by the same geometrical data. Its precise form is given by one half of the norm of certain Killing vector field, which allows a supersymmetric extension.

There are several directions to go from here. Our derivation relies heavily on the established properties of 1/4 BPS dyons and monopole moduli space. While there is little doubt that this is a valid derivation, it may be worthwhile to rederive our exact result from a different perspective. For instance, one may imagine deriving the exact bosonic potential from a point particle point of view. Another venue would be to find the supersymmetric low energy Lagrangian directly from the field theory, along the line of Gauntlett and Blum [14]. Naturally, we expect to recover the 1/4 BPS dyon spectra as quantum BPS states of this low energy dynamics. The actual form of the wavefunction is currently under investigation for the minimal case of $SU(3)$ [15].

Another interesting question concerns monopoles when the symmetry breaking is not maximal [16]. The gauge symmetry breaking is determined by both $\hat{b}\cdot\phi(\infty)$ and $\hat{a}\cdot\phi(\infty)$. If there is unbroken nonabelian gauge symmetry by $\hat{b}\cdot\phi(\infty)$, some of magnetic monopoles becomes massless. The moduli space acquires an enhanced isometry, corresponding to unbroken gauge groups. By $\hat{a}\cdot\phi(\infty)$, the unbroken gauge symmetry could remain unbroken or gets broken [17]. In the former case, the massless monopole clouds screens the color magnetic charge of massive monopoles, and the strength of static monopole-monopole force, say in the singlet channel, may be different from the naive expectation. Such a deviation has been observed in the large N context quite recently [18]. It would be quite interesting to quantize massless monopole motion and find the resulting quantum effective potential between massive monopoles. However, we should also point out that it is still unclear whether the moduli space dynamics is a valid approximation in the case of nonmaximal symmetry breaking.

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